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From the Editor-in-Chief

In this column, written by one of the occupants of the position of editor-in-chief and included in every volume whose number is divisible by twenty, we relate comments from authors and readers concerning papers that have recently appeared in *Linear Algebra and its Applications*. The column will contain errata, additional references, and historical and other comments that we believe will be of interest to readers of the journal.

(1) V. Nikiforov, Walks and the spectral radius of graphs, 418 (2006) 257–268. Yaoping Hou has pointed out to the author that in several instances in the paper "semiregular" must be replaced by "pseudo-semiregular". The following corrections from Nikiforov fix the problem.

(a) Theorem 5 should read as:

Theorem. *For every graph G,*

$$\mu^r(G) \geqslant \frac{w_{q+r}(G)}{w_q(G)} \tag{1}$$

for all $r > 0$ and odd $q > 0$.
If $q > 1$, equality holds in (1) if and only if each component of G has spectral radius $\mu(G)$ and is pseudo-regular or, if r is even, pseudo-semiregular.
If $q = 1$, equality holds in (1) if and only if each component of G has spectral radius $\mu(G)$ and is regular or, if r is even, semiregular.

Here $w_k(G)$ is the number of k-walks of G.

(b) Theorem 11 should read as:

Theorem. *Let $G = G(n)$ be a bipartite graph with eigenvalues $\mu_1 \geqslant \cdots \geqslant \mu_n$. If G is pseudo-semiregular, then for all $s \in [n]$ such that $0 < |\mu_s| < \mu(G)$ every eigenvector to μ_s is orthogonal to $\mathbf{i}$. If G is connected, the converse is also true.*

(c) The case of equality stated in Theorem 4 of Ref. [18] is correct.

(For more details the reader is referred to the corrected version of the paper in the arXiv.math.CO/0506259.)

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The question was posed as to whether, in case G is a connected bipartite graphs, equality holds in this inequality for every even $q \geqslant 2$ and $r \geqslant 2$. Lingsheng Shi has discovered a counterexample: Let P_4 be a path of length 3. Then $\rho(P_4) = 2\cos(\pi/5) = (1+\sqrt{5})^2/2$ and $w_2(P_4) = 16$. Therefore

$$\rho^2(P_5) = (1+\sqrt{5})^2/4 < 8/3 = w_3(P_4)/w_2(P_2).$$

In fact he has shown that for all $r > 0$ and even $q > 0$,

$$\rho^r(P_4) < w_{q+r}(P_4)/w_q(P_4)$$

and that for $r \geqslant 1$,

$$\rho^r(P_4) = \lim_{p\to\infty} w_{p+r}(P_4)/w_p(P_4).$$

(2) Y. Gao, Z. Li, Y. Shao, Sign patterns allowing nilpotence of index 3 (in press, available online). The authors would like to thank Michael Cavers for pointing out that the sign pattern

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

should be included in the list in Lemma 5.2. While this paper was in press, it came to the authors' attention that irreducible 3×3 sign patterns having at least one nonzero diagonal entry that allow nilpotence had been characterized in T. Britz et al., Minimal spectrally arbitrary sign patterns, SIAM J. Matrix Anal. Appl. 26 (2004) 257–271.

(3) C.K. Li, R. Mathias, Interlacing inequalities for totally nonnegative matrices, 341 (2002) 35–44. The authors have written to say that in the proof of Theorem 2.1, they reduce a given totally nonnegative matrix to a tridiagonal totally nonnegative matrix by some suitable similarity transformations. Hugo Woerdeman pointed out to them that the description at the top of p. 38 of the paper is not entirely clear and accurate. Here they present a more elaborated and accurate proof:

Suppose $A = (a_{ij})$ is an $n \times n$ totally nonnegative matrix. Eliminate the nonzero entries at the $(n, 1), (n-1, 1), \ldots, (3, 1)$ positions by elementary matrices $S_n, S_{n-1}, \ldots, S_3$, where for $j = n, n-1, \ldots, 3$, the matrix S_j is obtained from I_n by changing the $(j, j-1)$th entry to $-a_{j,1}/a_{j-1,1}$ if $a_{j,1} > 0$ (which implies that $a_{j-1,1} > 0$) and $S_j = I_n$ if $a_{j,1} = 0$. Then

$$A_1 = S_3 S_4 \cdots S_n A S_n^{-1} S_{n-1}^{-1} \cdots S_3^{-1}$$

is totally nonnegative and has zero entries at the $(n, 1), (n-1, 1), \ldots, (3, 1)$ positions. Suppose $n \geqslant 4$. Eliminate the nonzero entries of A_1 at the $(n, 2), (n-1, 2), \ldots, (4, 2)$ positions by elementary matrices $T_n, T_{n-1}, \ldots, T_4$. Then

$$A_2 = T_4 \cdots T_n A_1 T_n^{-1} \cdots T_4^{-1}$$

is totally nonnegative and has zero entries at the (i, j)th positions for $j = 1, 2$ and $n \geqslant i > j + 1$. If $n \geqslant 5$, apply the arguments to A_2 to eliminate the nonzero entries in the $(n, 3), \ldots, (5, 3)$ positions. Repeating these procedures, we will obtain a totally nonnegative matrix A_{n-2} with zero entries at the (i, j)th positions for $j = 1, \ldots, n-2$ and $n \geqslant i > j + 1$. Now, $B = A_{n-2}$ has the form SAS^{-1}, and we can apply the above arguments to B^t to get the desired tridiagonal totally nonnegative matrix which is similar to A.

(4) C. Draper, C. Martin, Gradings on g_2, 418 (2006) 85–111. The authors report: After writing the work we have learned that Y. Bahturin jointly with E. Zelmanov have obtained independently a classification of gradings on g_2.

(5) T. Ando, Löwner inequality of indefinite type, 385 (2004) 73–80. K. Nordström reports that: Lemma 3 of this paper, attributed to Smul'jan (1991) (Ref. [7]), can be found in K. Nordström, Some further aspects of the Löwner-ordering antitonicity of the Moore–Penrose inverse, Comm. Statist. Theory Methods 18 (1989) 4471–4489, where the result is shown to follow from the inertia equality:

$$\mathrm{In}(B^{-1} - A^{-1}) = \mathrm{In}(A - B) - (\mathrm{In}A - \mathrm{In}B),$$

valid for symmetric (selfadjoint) invertible matrices A and B. In that paper the result is also extended to symmetric singular matrices using the Moore–Penrose inverse.

Smul'jan discovered the result independently, extending it to bounded operators on a Hilbert space. This extension was in turn rediscovered independently by Hassi and Nordström (1992); see the discussion in Ref. [5].

(6) H.-K. Du, C.-Y. Deng, The representation and characterization of Drazin inverses of operators on a Hilbert space, 407 (2005) 117–124. J.J. Koliha has pointed out that this paper overlaps earlier papers by Djordjevic and Stanimirovic in Czech. J. Math. (51 (2001) 671–634) and Koliha in Glasgow Math. J. (38 (1996) 367–381),

(7) K. Yanagi, S. Furuichi, K. Kuriyama, On trace inequalities and their applications to noncommutative communication theory, 395 (2005) 351–359. S. Furuichi and K. Yanagi have produce a counterexample to a trace inequality proposed in that paper, namely

$$\mathrm{Tr}[(A+B)^s\{A(\log A)^2 + B(\log B)^2\} - (A+B)^{-1+s}(A\log A + B\log B)^2] \geqslant 0$$

for any s with $-1 < s \leqslant 1$ and positive matrices $A \leqslant I$ and $B \leqslant I$. For $0 \leqslant s \leqslant 1$ this inequality was proved in J.I. Fujii, A trace inequality arising from quantum information theory, 400 (2005) 141–146. They produce a counterexample for s in $(-1, 0)$ as follows:

Put $A = \exp^{-X}$ and $B = \exp^{-Y}$ for $X, Y > 0$. Then the inequality is equivalent to

$$\mathrm{Tr}[(\mathrm{e}^{-X} + \mathrm{e}^{-Y})^s(\mathrm{e}^{-X}X^2 + \mathrm{e}^{-Y}Y^2) - (\mathrm{e}^{-X} + \mathrm{e}^{-Y})^{s-1}(\mathrm{e}^{-X}X + \mathrm{e}^{-Y}Y)^2] \geqslant 0.$$

Taking

$$X = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}, \quad Y = \begin{bmatrix} 4 & 0 \\ 0 & 25 \end{bmatrix}, \quad s = -1/2$$

we get -0.441722.

(8) A. Klein, G. Mélard, T. Zahaf, Construction of the exact Fisher information matrix of Gaussian time series models by means of matrix differential rules, 321 (2000) 209–232. The authors write to make some corrections and amplifications deduced from the implementation and testing of the method:

The initialization stage described in Section 5, Remark 4, can fail. The whole p. 229, except the last sentence, should be replaced by the following:

Note that $\mathbb{E}\{w_1 \otimes w_1\} = \mathrm{vec}\, Q$. *Initial covariances involving only the process* $\tilde{x}_t$, *like* $\mathbb{E}\{\tilde{x}_1 \otimes \tilde{x}_1\}$ *or* $\mathbb{E}\{(\tilde{x}_1^\theta)^\mathrm{T} \otimes \tilde{x}_1\}$ *or* $\mathbb{E}\{\tilde{x}_1^\theta \otimes \tilde{x}_1^\theta\}$, *make use of the initial values discussed in Remarks* 1 *and* 2. *For example* $\mathbb{E}\{\tilde{x}_1 \otimes \tilde{x}_1\} = P_{1|0}$. *Note that the process* x_t *is stationary and does not depend on the observations made at times* $t = 1, \ldots, N$.

In order to have a stable solution for all the equations involving the process x_t, *we can run the recurrence equations of Section* 4 *for* $t = 1, \ldots, N_0$, *where* N_0 *is large enough, watching for convergence. This needs initial values which can be any non-zero matrices of appropriate sizes. When we have the stable solutions, we use them as the initial values of the recurrence equations of Section* 4 *but for* $t = 1, \ldots, N$, *where* N *is the number of observations.*

It can be seen that the original initialization $x_1 = Fw_0$ is not compatible with $\mathbb{E}\{\tilde{z}_1\tilde{z}_1^\mathrm{T}\} = B_1$ and is therefore wrong. For example, for a univariate AR(1) process (i.e. the case where $m =$

$1, p = 1, s = 0$) described by the equation $z_t = \theta_1 z_{t-1} + w_t$ with $Q = 1$, the state vector is $x_t = \theta_1 z_{t-1}$ and its variance is $\theta_1^2/(1-\theta_1^2)$. When we replace Φ and F by θ_1 in (65) we obtain effectively $\theta_1^2/(1-\theta_1^2)$ as a stable solution of that equation. However, following the wrong original approach with the stated initial value $\mathbb{E}[x_1 \otimes x_1] = (F \otimes F)\operatorname{vec} Q = \theta_1^2$, we have of course a time-dependent solution which is not appropriate. This illustrates the mistake. Another solution is possible but it would involve $P_{1|0}$ which is not fully computed here and similar matrices for expectations involving derivatives $\tilde{x}_1^\theta$.

A related paper [a] contains a discussion of the implementation in the MATLAB environment as well as numerical aspects related to it. Note that computing the derivatives of the autocovariances is the subject of another paper [b] where some algebraic aspects related to the problem are mentioned.

[a] A. Klein, G. Mélard, J. Niemczyk, T. Zahaf, A program for computing the exact Fisher information matrix of a Gaussian VARMA model, submitted for publication.
[b] J. Niemczyk, Computing the derivatives of the autocovariances of a *VARMA* process, in: J. Antoch (Ed.), COMPSTAT'2004, 16th Symposium held in Prague, Czech Republic, Proceedings in Computational Statistics, Physica-Verlag, Heidelberg (2004), pp. 1593–1600.

Some minor corrections are also needed.

(1) In (48), last term, replace I_n by I_m.
(2) In (54), line 3, replace F^{T} by $\overline{F}_t^{\mathrm{T}}$.
(3) Also in (54), last term, replace $M_{mn,n}$ by $M_{mn,m}$.
(4) In (58), first line, replace M_{n,n^2} by $M_{n^2,n}$.
(5) On p. 229, 10 occurences of vecF^θ should be replaced by F^θ but most of these relations are useless, because of the changes to that page.

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